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# DIRECT NUMERICAL SIMULATION OF TURBULENT BOUNDARY LAYERS WITH ADVERSE PRESSURE GRADIENTS

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## Abstract

Direct numerical simulations of the Navier-Stokes equations have been carried out with the objective of studying turbulent boundary layers with adverse pressure gradients. In the literature two approaches to the analysis of the equation of motion are found. In this work we conclude that a nonlinear theory is required for relating the freestream variation to other meanflow parameters. Comparison of turbulent statistics from the zero pressure gradient case and the two adverse pressure gradient cases show the development of a second peak in the turbulent energy in agreement with experiment. The turbulent mean flows have also been investigated using a differential Reynolds-stress model. The model predictions are compared with the direct simulations and low Reynolds number effects are investigated.

## 1 Introduction

The analysis of adverse pressure gradient (APG) turbulent boundary layers has been going on for a long time, but only in recent years have direct numerical simulations (DNS) of these flows become possible, albeit for low Reynolds numbers.

The conditions needed for self-similarity as well as for the onset of separation have been the subject of several investigations. Clauser<sup>[2]</sup> performed experiments where he adjusted the pressure gradient such that a self-similar turbulent boundary layer was obtained. A constant nondimensional pressure gradient was shown to be the condition for self-similarity. Mellor & Gibson<sup>[10]</sup> showed that self-similarity is obtained if  $U \sim x^m$ . Bradshaw<sup>[1]</sup> measured three self-similar turbulent boundary layers with  $m = 0, -0.15, -0.255$  and concluded that the last one was near separation. Head<sup>[5]</sup> used an integral method to calculate the turbulent boundary layer for  $m = -0.15, -0.255, -0.35$  and concluded that the solution is unique in the first case but not in the second one while the third one has no solution. Skåre and Krogstad<sup>[13]</sup> did experiments on turbulent boundary layers near separation. Their results showed that the shape factor approaches 2 when  $m = -0.22$ .

DNS of a zero pressure gradient turbulent boundary layer was done by Spalart<sup>[14]</sup>. Spalart & Leonard<sup>[16]</sup> did also DNS of self-similar APG turbulent boundary layer using a similarity coordinate system. In these simulations the shape factor approaches 2.3 and  $m = -0.22$  near separation. Spalart & Watmuff<sup>[17]</sup> compared experiments and DNS of an APG turbulent boundary layer in a varying pressure gradient and they found good agreement. Coleman & Spalart<sup>[3][15]</sup> performed DNS of a separation bubble with heat transfer included. Na & Moin<sup>[11]</sup> have recently performed DNS of a separation bubble.

The consensus from these investigations seems to be that a power law freestream velocity is a requirement for self-similarity and that separation occurs for about  $-0.25 < m < -0.20$  with a shape factor of about 2.

## 2 Direct Numerical Simulations

DNS of the Navier-Stokes equations have been carried out with the objective of studying APG turbulent boundary layer flows. The code used for the DNS is developed at KTH and FFA<sup>[8][9]</sup>.

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The program uses spectral methods with Fourier discretization in the horizontal directions and Chebyshev discretization in the normal direction. Since the boundary layer is developing in the downstream direction, it is necessary to use non-periodic boundary conditions in the streamwise direction. This is possible while retaining the Fourier discretization if a fringe region is added downstream of the physical domain. In the fringe region the flow is forced from the outflow of the physical domain to the inflow. In this way the physical domain and the fringe region together satisfy periodic boundary conditions. The fringe region is implemented by the addition of a volume force which form is designed to minimize the upstream influence. Time integration is performed using a third order Runge-Kutta method for the advective and forcing terms and Crank-Nicolson for the viscous terms.

The simulations start with a laminar boundary layer at the inflow which is tripped by a random volume force near the wall. All the quantities are non-dimensionalized by the freestream velocity ( $U$ ) and the displacement thickness ( $\delta^*$ ) at the starting position of the simulation ( $x = 0$ ) where the flow is laminar. At that position  $Re_{\delta^*} = 400$ . The length (including the fringe), height and width of the computation box were  $450 \times 24 \times 24$  in these units. The number of modes was  $480 \times 161 \times 96$ , which gives a resolution in plus units of  $\Delta X^+ = 16$  and  $\Delta Z^+ = 4.3$ . The useful region was confined to  $x = 150 - 350$  which corresponds to  $Re_{\delta^*} : 550 - 1200$  or  $Re_{\Theta} : 330 - 700$ .

The simulations were run for a total of 4500 time units ( $\delta^*/U$ ), and the sampling for the turbulent statistics was performed during the 2000 last time units.

It was verified that the accuracy of the DNS and its statistics was sufficient by repeating the computation on a coarser resolution ( $320 \times 101 \times 64$  modes) and with a shorter averaging time (1000 time units).

### 3 Results from Analysis

The equation describing the outer part of an incompressible turbulent boundary layer is given by

$$u \frac{\partial}{\partial x} u + v \frac{\partial}{\partial y} u = -\frac{1}{\rho} \frac{dP}{dx} - \frac{\partial}{\partial y} \langle uv \rangle, \quad (1)$$

where  $u$  is the mean streamwise velocity,  $v$  the mean wall normal velocity,  $\frac{dP}{dx}$  the pressure gradient and  $\langle uv \rangle$  the Reynolds stress.

To find a self-similar solution we want the equation to be independent of  $x$ , i.e. we want to find solutions in the form

$$(u - U)/u_\tau = F(\eta), \quad -\langle uv \rangle/u_\tau^2 = R(\eta) \quad (2)$$

where

$$\eta = y/\Delta(x), \quad \Delta = U\delta^*/u_\tau. \quad (3)$$

Introducing  $F(\eta)$  and  $R(\eta)$  into the equation (1) gives an equation which can be written

$$-2\beta F - (1 + 2\beta)\eta \frac{dF}{d\eta} = \frac{dR}{d\eta}, \quad (4)$$

when  $\frac{u_\tau}{U} \rightarrow 0$ . The classical treatment of the equations which involves outer and inner equations and a matching of the solutions, leads to the logarithmic friction law, implying that  $\frac{u_\tau}{U} \rightarrow 0$  in the limit of very high Reynolds number. This is utilized in the derivation of equation (4), shown in Tennekes & Lumley<sup>[19]</sup>, Mellor & Gibson<sup>[10]</sup> and Henkes<sup>[6]</sup>.

From equation (4) one obtains a condition for self similarity as a parameter  $\beta$  that should be constant

$$\beta = \frac{\delta^*}{\tau_w} \frac{dP}{dx}. \quad (5)$$

$\beta = \text{constant}$  can be seen as a balance of the forces acting on the boundary layer; skin friction and pressure gradient, as argued by Clauser<sup>[2]</sup>.

Using the argument  $\frac{u_\tau}{U} \rightarrow 0$  in the analysis of the integrated momentum equation,

$$\left(\frac{U}{u_\tau}\right)^2 \frac{d}{dx} \Theta - \frac{2\beta}{H} = 1 + \beta, \quad (6)$$

Tennekes & Lumley<sup>[19]</sup> perform a linearization. The linearized version of equation (6) is

$$\frac{U}{u_\tau^2} \frac{d}{dx} (\Delta u_\tau) = 1 + 2\beta. \quad (7)$$

The linearization is equivalent to the assumption that the shape factor,  $H \equiv \frac{\delta^*}{\Theta}$  is equal to one.

By keeping the ratio  $\frac{u_\tau}{U} \approx \text{constant}$  in equation (7) Tennekes & Lumley obtain a relation between  $m$  and  $\beta$  which reads

$$m = -\frac{\beta}{1 + 3\beta} \quad (8)$$

with

$$U \sim x^m \quad (9)$$

In the following, the linearization will be replaced by a less restrictive approximation to derive a relation between  $m$ ,  $\beta$  and  $H$ . The full integrated momentum equation (6) can be written

$$\frac{U' \delta^*}{U \delta^{*'}} = -\frac{\beta}{H(1 + \beta) + 2\beta + (H - 1)\beta(1 - \frac{u_\tau' U}{u_\tau U'})} \quad (10)$$

with the aid of the relation

$$H = \frac{1}{1 - \frac{u_\tau}{U} G} \quad (11)$$

where

$$G = \int_0^\infty F^2 d\eta. \quad (12)$$

$G$  must be constant if the boundary layer is to be self similar, i.e.  $F$  does not change its shape. From equation (11) the limit  $H \rightarrow 1$  as  $\frac{u_\tau}{U} \rightarrow 0$  is consistent with the linearization described above. Since the logarithmic function grows very slowly when the argument is large, a better assumption than  $\frac{u_\tau}{U} \rightarrow 0$  for moderately high Reynolds numbers is that  $\frac{u_\tau}{U} \approx \text{constant}$ . If  $\frac{u_\tau}{U}$  is kept constant, which means that

$$\frac{u_\tau' U}{u_\tau U'} = 1 \quad (13)$$

equation (10) can be integrated. Together with the definition of  $\beta$ , equation (5), which can be written

$$\beta = -\frac{\delta^*}{u_\tau^2} U \frac{dU}{dx}, \quad (14)$$

equation (10) gives

$$U \sim x^m, \quad \delta^* \sim x. \quad (15)$$

with

$$m = -\frac{\beta}{H(1+\beta) + 2\beta} \quad (16)$$

Equation (8) is recovered from equation (16) by letting  $H = 1$ .

In experiments (Skåre and Krogstad<sup>[13]</sup>) with high pressure gradients where flows near separation has been investigated, one has observed that  $H \rightarrow 2$ . If  $\beta \rightarrow \infty$  equation (16) reduces to  $m = -\frac{1}{H+2}$ . If now  $H \rightarrow 2$ ,  $m$  becomes  $-0.25$  which is close to the values reported from the experiments and DNS of turbulent boundary layers near separation, (see e.g. Stratford<sup>[18]</sup> and Spalart & Leonard<sup>[16]</sup>). Comparison with the linear theory, which gives a value of  $m = -\frac{1}{3}$  when  $\beta \rightarrow \infty$ , shows that the linearization, which might be correct for very high Reynolds number, is an insufficient theory for low and moderate Reynolds number flows.

Equation (10) can be integrated when assuming a constant right hand side. This done by Mellor & Gibson<sup>[10]</sup> and gives

$$U \sim \tilde{x}^m \quad (17)$$

with

$$\tilde{x} = \int_0^x \frac{u_\tau U_0}{u_{\tau 0} U} dx \quad (18)$$

where the subscript 0 represents initial values at  $x = 0$  and where  $m$  in their case is equal to the right hand side of equation (10). With  $\frac{u_\tau}{U} = \text{constant}$  the variable  $\tilde{x}$  in equation (17) becomes  $x$  and the results in equations (15) and (16) are recovered. In experiments and DNS, (see references in the Introduction),  $H$  and  $\frac{u_\tau}{U}$  are far from their asymptotic values of 1 and 0 respectively. This is due to the slow increase of  $\ln(Re)$  with  $Re$ .  $H$  and  $\frac{u_\tau}{U}$  are on the other hand fairly constant for the same reason. It seems as the linearized theory is not applicable to real flows (finite  $Re$ ) and that equation (17) does not give more information than the nonlinear theory with a constant  $\frac{u_\tau}{U}$  as an approximation. The agreement between equation (16) and data from measurements and DNS is shown in the table below.

Case	$\beta$	$H$	$m$	$m = -\frac{\beta}{H(1+\beta)+2\beta}$	$m = -\frac{\beta}{1+3\beta}$
APG1	0.24	1.60	-0.077	-0.097	-0.14
APG2	0.67	1.63	-0.15	-0.16	-0.22
Bradshaw 1	0.9	1.4	-0.15	-0.20	-0.24
Bradshaw 2	5.4	1.54	-0.255	-0.26	-0.31
Skåre & Krogstad	20	2.0	-0.22	-0.24	-0.33
Spalart & Leonard	1.8	1.65	-0.21	-0.22	-0.28
	8.0	1.92	-0.23	-0.24	-0.32
	$\infty$	2.3	-0.22	-0.23	-0.33

Table 1: Comparison of  $m$  from the nonlinear/linear theory.

## 4 Results from DNS

Results from two direct numerical simulations of APG turbulent boundary layers as well as one zero pressure gradient case (ZPG) are presented. The pressure gradient in the first APG case (APG1)

is close to that for which the corresponding laminar boundary layer separate and in the second (APG2) the same pressure gradient as in Bradshaw's first experimental APG case<sup>[1]</sup>. The pressure gradient is applied through the freestream velocity,  $U$ , which is described by a power law,  $U \sim x^m$ . For APG1  $m = -0.077$  and for APG2  $m = -0.15$  which corresponds to  $\beta \approx 0.24$  and  $\beta \approx 0.67$  respectively. The results from the ZPG are taken from a simulation by Henningson & Lundbladh<sup>[7]</sup>. This simulation is not as highly resolved as the other two and the computational box height is not large enough, see the ZPG-profile in Figure 2b. The results are included only as a comparison to the two APG simulations.

The  $\beta$ -parameter is shown for the two APG cases in Figure 1a, and  $H$  as a function of  $Re_\Theta$  for all three cases in 1b. The  $\beta$ -parameter is close to constant and the shape factor varies slower for higher pressure gradients. In Figure 2a the skin friction,  $C_f$ , is shown.  $C_f$  is lower for higher pressure gradients and would become zero at separation. In Figure 2b Reynolds normal stress at the same position for the three cases are shown. The tendency towards a second peak in the  $u_{rms}$  profile for higher pressure gradients agrees with the experimental results from Nagano, Tagawa & Tsuji<sup>[12]</sup>.

As the  $\beta$ -parameter is constant we might expect a self-similar boundary layer. From the velocity profiles at positions downstream (Figure 3a) it is difficult to draw any conclusions about self similarity since the Reynolds number variation is small. The profiles for the Reynolds stress (Figure 3b) show clearly that self similarity is not obtained since the maximum grows downstream. This is due to low Reynolds number, as we will see in the next section, where also further analysis of higher order statistics and comparison with model predictions are made.

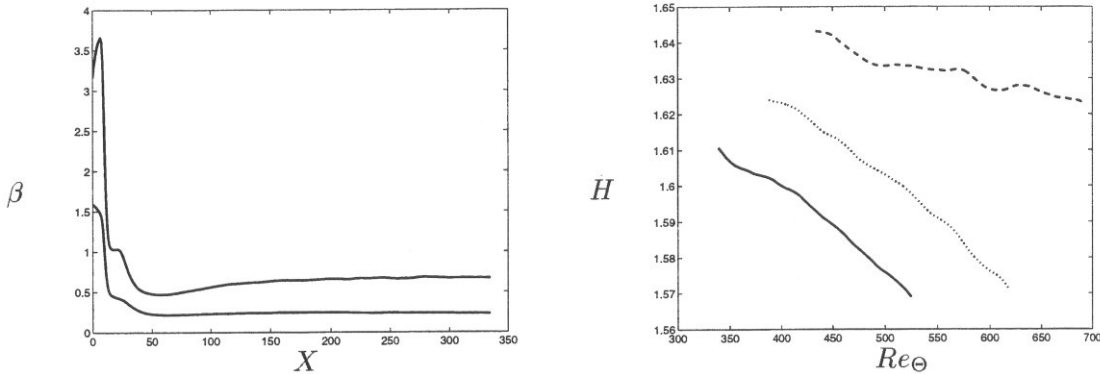


Figure 1: a)  $\beta$  for the two APG cases. b)  $H$  — ZPG;  $\cdots$  APG1;  $--$  APG2

## 5 Comparison between calculations with turbulence models and DNS

Calculations with the Differential Reynolds Stress Model (DRSM) of Jakilić, Hadžić and Hanjalić<sup>[4]</sup> were performed at low Reynolds numbers to compare with data from our DNS. Profiles for the velocity and turbulence obtained from the DNS at  $x = 150$  were used as initial data. Comparison was made at  $x = 335$  for APG2, where  $Re_\Theta = 690$ . The DRSM calculations show that transients are dominating in the beginning of the calculations but at the point of comparison with the DNS-data the solution is not sensitive to small changes in initial data. Thus the comparison is meaningful since the difference between the model predictions at low and high Reynolds number are due to the dependence on the Reynolds number and not to the influence of the initial conditions.

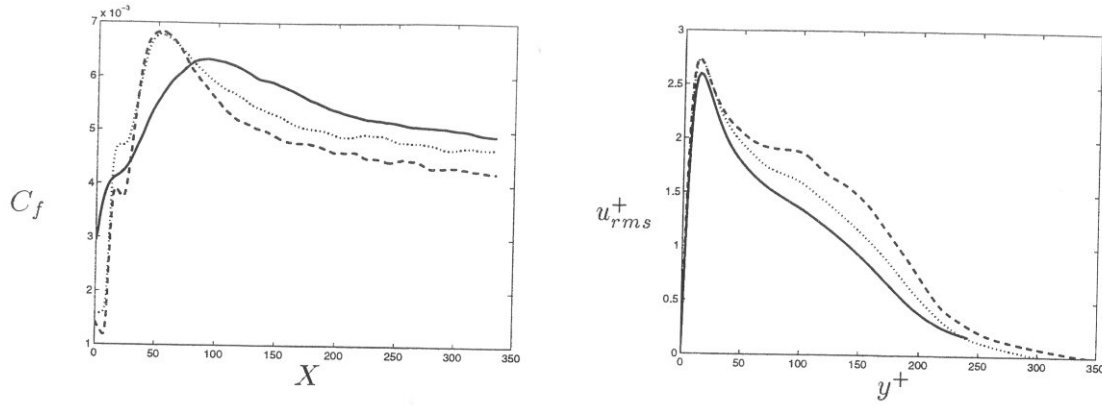


Figure 2: a)  $C_f$ . b)  $u_{rms}$  at  $300\delta^*$  — ZPG;  $\cdots$  APG1;  $--$  APG2

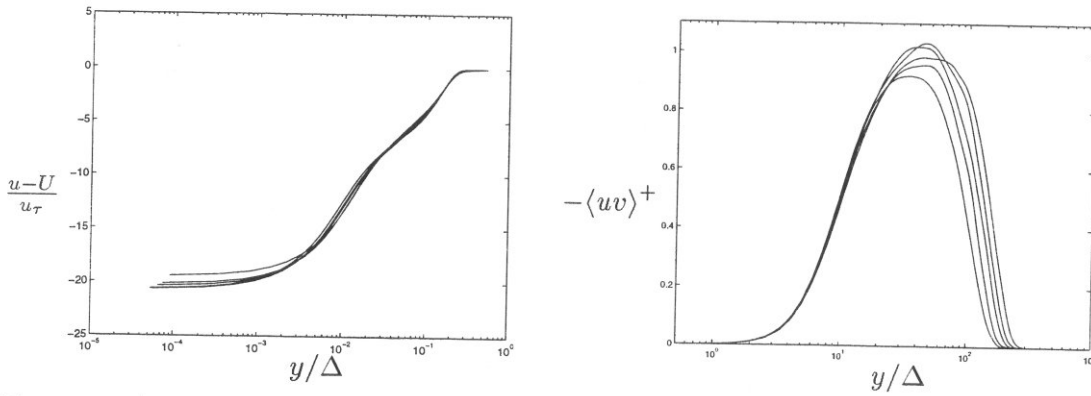


Figure 3: a) Velocity profiles for APG1. b)  $\langle uv \rangle^+$  profiles for APG1 at  $150 - 350\delta^*$  downstream.

Figures 4a,b show close agreement for the velocity profile in inner and outer scalings at  $Re_\theta = 690$  and  $\beta \approx 0.67$ , as computed with the DNS and DRSM. The figure also shows the large- $Re$  similarity state for the DRSM. Only a small logarithmic part in the inner layer is found at  $Re_\theta = 690$ , due to the low Reynolds number. The streamwise Reynolds normal stress is compared in Figures 5a,b. The results are shown in both inner and outer layer scalings, and the similarity solution for the DRSM is included as well. Differences between the solution at  $Re_\theta = 690$  and the similarity solution are significant. The results with the DRSM closely agree with the DNS at  $Re_\theta = 690$ , showing that the DRSM reproduces the physics of APG boundary layers at relatively low Reynolds numbers. The peak in the Reynolds normal stress in the DNS and DRSM at  $Re_\theta = 690$  is part of the inner layer, but there already is a tendency to develop a second peak in the outer layer, which indeed has been established in the similarity solution with the DRSM.

## 6 Conclusions

DNS of the Navier-Stokes equations has been carried out with the objective of studying APG turbulent boundary layers. The pressure gradient parameter is found to be constant when the freestream velocity varies according to a power law. An analysis of the equations describing the meanflow based on the approximation of a constant ratio of the freestream velocity to the friction velocity is presented. The analysis leads to a relation between the power law and both the pressure

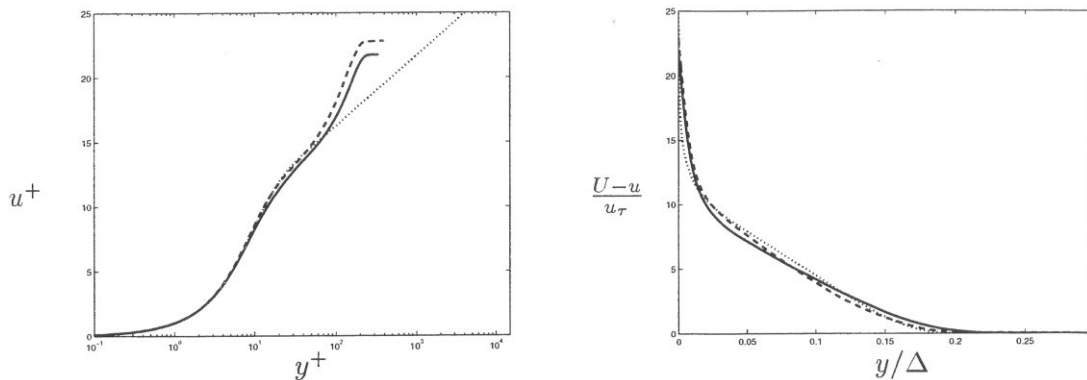


Figure 4: Velocity profiles for APG2 in a) inner scaling b) outer scaling — DNS; - - DRSM; ... Asymptotic

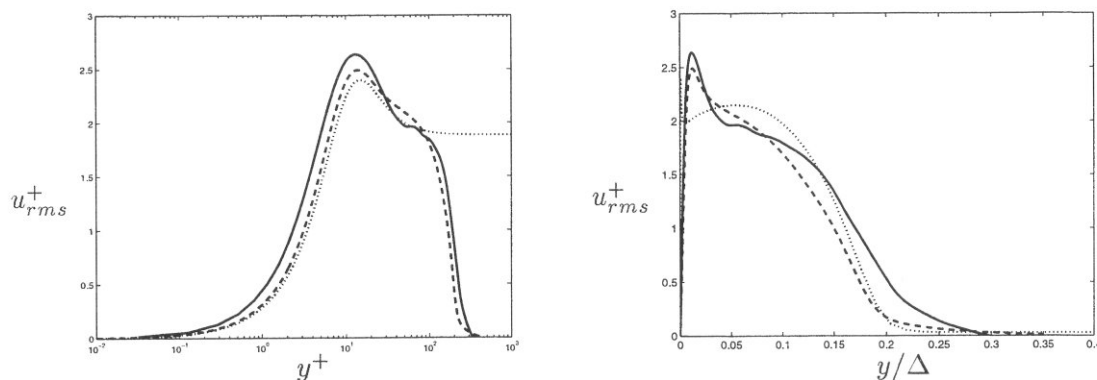


Figure 5: a)  $u_{rms}^+$  for APG2 in a) inner scaling b) outer scaling — DNS; - - DRSM; ... Asymptotic

gradient parameter and the shape factor. This relation gives better agreement than the linearized analysis based on a ratio of the freestream velocity to the friction velocity of zero. The linearization is equivalent to the assumption that the shape factor is equal to one.

Comparison of turbulent statistics from the ZPG and the two APG cases show the development of a second peak in the turbulent energy in agreement with experiment.

A differential Reynolds-stress model was used to predict the mean flow. Comparison with the direct simulations showed that low Reynolds number effects are well captured.

The work presented in Section 5 was carried out together with Dr. Ruud Henkes whom we wish to thank.

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