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Kolmogorov spectrum consistent optimization for multi-scale flow decomposition

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Multi-scale analysis is widely adopted in turbulence research for studying flow structures corresponding to specific length scales in the Kolmogorov spectrum. In the present work, a new methodology based on novel optimization techniques for scale decomposition is introduced, which leads to a bandpass filter with prescribed properties. With this filter, we can efficiently perform scale decomposition using Fourier transform directly while adequately suppressing Gibbs ringing artifacts. Both 2D and 3D scale decomposition results are presented, together with qualitative and quantitative analysis. The comparison with existing multi-scale analysis technique is conducted to verify the effectiveness of our method. Validation of this decomposition technique is demonstrated both qualitatively and quantitatively. The advantage of the proposed methodology enables a precise specification of continuous length scales while preserving the original structures. These unique features of the proposed methodology may provide future insights into the evolution of turbulent flow structures. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4871106]

I. INTRODUCTION

Turbulent flow consists of self-similar structures with a wide range of length scales. This selfsimilarity has led to the idea of energy cascade,^{1–3} which governs turbulent flows universally. Over the past few years, multi-scale decomposition has gained increasing interest in turbulence community for both modeling and analysis,⁴ and has proven to be useful for understanding the evolution of eddies and the interaction between turbulent flow structures at different scales. This can be pivotal in understanding the morphology and dynamics of turbulent flow structures. An exploration in this direction can be motivated by the question of whether there are universal structures across different scales.⁵

The energy cascade phenomenon for a turbulent flow is characterized by the Kolmogorov spectrum which gives the variation of energy E(k) contained by all eddies with different length scales (k) in a log-transformed Fourier space. A *scale* for a turbulent flow is usually referred to as a range of wavenumbers that can be obtained via a perfect bandpass filter (BPF) in the Kolmogorov spectrum, which also corresponds to a perfect BPF in Fourier space. However, it is well known that Fourier transform with perfect BPF usually produces strong Gibbs ringing artifacts⁶ due to insufficient basis.

This phenomenon can generate spurious structures, leading to error-prone conclusions regarding the flow characteristics. The drawback of using perfect BPF in Fourier space has motivated the research in the direction of utilizing local-support basis multi-resolution methods such as *wavelets*^{7,8} and *curvelets*.⁹ Wavelets are based on a symmetric local basis, while curvelets, as an extension of wavelets, have an extended dimension of localized orientation with finer-scale ridge-shaped basis functions.

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In recent years, curvelets have found increasing use among various research groups for scale decomposition studies. Bermejo-Moreno and Pullin¹⁰ presented multi-scale geometrical decomposition for isotropic flow and performed characterization of the flow structures based on direct numerical simulation (DNS) data. Geometry of enstrophy and dissipation structures were shown by Bermejo-Moreno *et al.*¹¹ Ma *et al.*^{12,13} also used curvelets to provide a geometric analysis of flow structures. Using a similar methodology, Yang *et al.*¹⁴ showed the evolutionary geometry of the Lagrangian scalar field for stationary isotropic turbulence. Yang and Pullin¹⁵ reported results for anisotropic channel flow with a study of the geometry of Lagrangian and Eulerian structures.

In addition to wavelets and curvelets, Leung *et al.*¹⁶ also presented multi-scale decomposition based on spatial filtering. However, the effect of spatial filtering in Fourier space does not characteristically confine the results to a sharp band and hence a large overlap of different scales results as a consequence. Due to the large overlap, the flow structures cannot be uniquely identified as belonging to different energetic bands.

With the use of curvelets/wavelets, one can significantly reduce Gibbs rings due to band-limiting basis in comparison to the Fourier transform. However, a severe drawback with those methods is that they impose a strong restriction on the selection of scale location and bandwidth, leading to dyadic discontinuous scales with fixed bandwidth.

In fact, curvelets/wavelets were not originally designed for scale decomposition purpose. Their advantage of using a localized basis suits better representation of sharp edges and hence is a better way for data representation and compression. However, in the case of turbulence, scale decomposition is a global operation, and Kolmogorov spectrum is nothing but a symmetric extension of the Fourier transform. Thus, it is more discernible to perform scale decomposition directly in Fourier space. Moreover, by carefully examining the curvelets/wavelets scale decomposition results, they are no different from a BPF in Kolmogorov spectrum with a certain fall-off. This motivates us to develop a scale decomposition technique directly with Fourier transform. The distinct advantage of methods in Fourier space, e.g., *scale selection* and *bandwidth*, can be better exploited using this approach, as it enables us to have a continuous scale decomposition which is very important for more comprehensive study of coherent structures across different scales in turbulence.

The only hurdle which prevents most researchers from using this naive approach is the ringing artifacts due to Gibbs phenomena. However, if nearby scales are introduced using the global-support Fourier basis, it can significantly reduce Gibbs rings to similar levels as methods using a local-support basis. Considering the drawbacks of present methodologies, we propose an alternative to curvelets/wavelets, which in essence is an optimization-based approach to retaining the benefits of using Fourier transform and reducing Gibbs phenomena without having to confine the scale numbers to be only dyadic. Therefore, a filter in Fourier space can be directly designed to achieve multi-scale decomposition similar to ones using a local-support basis. The filtered data are then converted back to the physical domain, which gives us the scale decomposition results. Guided by the previous discussion on the advantages of using Fourier-based methods and drawbacks of curvelets/wavelets, our proposed method provides the following contributions to the field of multi-scale flow analysis:

- We developed a new scale decomposition method, Kolmogorov Spectrum Consistent Optimization (KoSCO), where the filter is designed directly in the Kolmogorov spectrum consistent space.
- We developed a novel optimization-based framework where an objective function is designed and its minimization gives the desired filter shape for scale decomposition.
- More flexible control over the band location and bandwidth in Kolmogorov spectrum is achieved with the proposed method. This unique feature of having continuous scales, which is the deficiency of the current methods based on curvelets/wavelets, can be beneficial for turbulence research as it greatly enhances the capability to track the evolution of structures across different continuous scales.

This paper is organized as follows. In Sec. II, a formal derivation of the proposed scale separation method and the solution procedure are given. In Sec. III, the method is verified for different test

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cases and the results are further discussed. Finally, in Sec. IV, we draw the conclusion and discuss the future work.

II. KOLMOGOROV SPECTRUM CONSISTENT OPTIMIZATION

In Secs. II A and II B, we point out some of the key observations which led us to the proposed method in this paper.

A. Filter design

First, we define the *scale* for a turbulent flow suitable for decomposition purposes. The Kolmogorov spectrum, which illustrates the energy cascade governing turbulent flows, consists of the total energy carried by different sizes of eddies. Ideally, a *scale* is referred to as being a perfect BPF in Kolmogorov spectrum, which is illustrated in Fig. 1(a) as the solid line. However, a perfect BPF exhibits strong Gibbs ringing artifacts, which will be further discussed in detail in Sec. III A.

Since Gibbs ringing artifacts are generated due to insufficient basis, they can be suppressed by adding a few neighboring scales. On the other hand, the number of neighboring scales should be kept at minimum in order to preserve the sharpness of the filter. Thus, by introducing a certain amount of nearby scales, the Gibbs rings can be suppressed, while the extent of the deviation from a perfect BPF is limited. The filtered results using such filter could therefore be much more meaningful than either a perfect BPF or a filter that includes an excessive amount of other scales.

Hence, the filter can be designed such that it has a fall-off from a perfect BPF to include some nearby scales. A fall-off is a non-zero extended region from the perfect BPF, which should be monotonic. Such monotonicity criterion is to ensure we do not introduce falsely magnified nearby scales; otherwise some spurious oscillating structures may appear. Thus, we model the filter with the fall-off as a Gaussian function

$$\hat{\mathcal{G}}(k) = \begin{cases} 1 & \text{if } k \in \beta \\ e^{-r^2/2\sigma^2} & \text{if } k \notin \beta \end{cases},$$
(1)

where β is the scale bandwidth; σ is a parameter to control the extent of the fall-off; and *r* is the distance from the perfect BPF. Fig. 1(a) illustrates the shape of the designed filter with a dashed line. This filter is applied to a Kolmogorov spectrum consistent space, which is obtained by using the similar procedure as in Kolmogorov spectrum calculation. This involves reduction of an N-dimensional field to a scalar (i.e., energy) in the Fourier space at different wavenumbers.



FIG. 1. Constraints for the filter design process. (a) Design of filter. Solid line: perfect bandpass filter; Dashed line: our filter. (b) Objective function and its components. $(-) E_i$ $(\cdots) E_r$ or E_g ; $(- \cdot -) E_k$.

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Now the problem is on how to specify an appropriate value of σ such that Gibbs ringing artifacts can be greatly suppressed while introducing a minimum amount of nearby scales. If σ is too small, Gibbs ringing artifacts will not be suppressed, whereas the filter remains very sharp. On the other hand, if σ is large, a wide range of nearby scales are added; although Gibbs ringing is suppressed, the notion of a sharp bandpass filter is lost. Thus, an optimization problem can be defined where we try to balance the preservation of the sharp filter fall-off and the removal of Gibbs rings by finding a suitable σ . We refer to such optimization as KoSCO and we entail further details in the following.

B. Objective function definition

Let \mathcal{G} be the desired filter and \mathcal{G}_0 be a perfect BPF ($\sigma \to 0$) defined using Eq. (1) in a Kolmogorov spectrum consistent space. Then, we can perform an inverse log transform and symmetric dimension extension to get the desired filter in Fourier space. In the following, *u* denotes the magnitude of the velocity vector **u**. All quantities with a hat ([^]) are Fourier transform of the respective quantities.

Our objective function consists of two terms. We define the first term as the difference to measure the closeness between the result from a perfect BPF and the result from the designed filter. This is to ensure that the desired filter has a sharp fall-off. Mathematically, this term can be written as

$$E_{k} = \psi \int \left(\hat{\mathcal{G}}^{2} - \hat{\mathcal{G}}_{0}^{2}\right) |\hat{u}|^{2} dk \quad \text{and} \quad \psi = \left(\int \hat{\mathcal{G}}_{0}^{2} |\hat{u}|^{2} dk\right)^{-1}, \quad (2)$$

where ψ is a normalization constant. E_k is a decreasing function (see Fig. 1(b)) with respect to the filter parameter σ defined in Eq. (1), since as σ decreases, the desired filter approaches a perfect BPF.

Next, we try to model the Gibbs rings or oscillation artifacts from which most of the decomposition methods (including wavelets and curvelets) suffer inherently. An example of this phenomenon is shown in Fig. 2 (which will be described further in Sec. III A). Applying a naive bandpass filter using FFT results in spurious rings which contaminates the results and is a known problem due to which Fourier based methods are not generally utilized for such decomposition purpose. We define the second term in our objective function as a measure of these spurious structures. Quantifying the Gibbs rings in the objective function allows us to reduce them as much as possible through our optimization process. This is the key to utilizing Fourier based methods and their superior advantages of continuous scale location and bandwidth. One implicit condition which the decomposition method must obey, and which is also physically intuitive, is that a region which is originally smooth (small gradients with low frequency) should also be smooth in the resulting decomposition. Based on this, we define the second term of our objective term as the *regularization* term for measuring the strength of the Gibbs rings, which measures the difference of the gradients in the original field and the decomposed result as a weighted sum of the gradients where the weight is inversely proportional to the strength of gradient and can be mathematically written as

$$E_r = \phi \sum_{\alpha} \int |w_{\alpha} : \partial_{\alpha} * u(1 - \mathcal{G})|^2 \quad \text{and} \quad \phi = \left(\sum_{\alpha} \int |w_{\alpha} : (\partial_{\alpha} * u)|^2 dk\right)^{-1}, \quad (3)$$



FIG. 2. Gibbs ringing phenomena. (a) Original, (b) Bandpass FFT, (c) Curvelets, (d) KoSCO.

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where * is the convolution operator; α denotes any derivative direction; : represents element wise product; ϕ is the normalization constant, and $w_{\alpha} = e^{-(\partial_{\alpha} u)^2/2\theta^2}$ is a Gaussian weight with respect to gradients of the original field. The parameter θ is the standard deviation of the gradients of the original field (calculated by comparing the value at each point with the mean gradient over the field). Although a Gaussian weight has been chosen for the present case, any function with a monotonic fall-off can be used to define the weights. We use the weight at different locations to control the influence of the regularization term, where regions with large gradients will have less impact on the measurement and vice versa. This will enforce a strong constraint only in smooth regions. Since the Fourier transform has global support basis, the constraint in smooth regions will also give the constraint in the surrounding non-smooth regions. Therefore, this term can effectively measure the overall strength of ringing artifacts. As shown in Fig. 1(b), with lower σ , E_r increases, which indicates that the result contains more rings. Note that the choice of the weight w_{α} only alters the objective function, and thus does not violate the invariance of spectral cascade rate.

Note that the two terms E_k and E_r are defined in two different domains: E_k in wavenumber domain and E_r in spatial domain–which is inefficient for optimization. Thus, we unify the two domains by transforming term E_r into wavenumber domain using Plancherel's theorem, which gives

$$E_g = \phi \sum_{\alpha} \int \left| \hat{w}_{\alpha} * \left[\hat{\partial}_{\alpha} \hat{u} (1 - \hat{\mathcal{G}}) \right] \right|^2 \mathrm{d}k.$$
(4)

 E_r and E_g have the same value; they only differ in space for optimization purpose. To enhance the order of magnitude of the difference measurement, both E_k and E_g are raised to an exponent *m*. In the remainder, *m* is chosen to be around 5 in the results presented here.

The total objective function can be defined as a linear combination of E_k and E_g as

$$E = (1 - \lambda)E_k^{\ m} + \lambda E_g^{\ m},\tag{5}$$

where λ is a regularization parameter in the range [0, 1] to control the relative importance of E_k and E_g . Not all λ values yield an optimizable problem, i.e., a minimizable function *E*. See Fig. 1(b) for an illustration of E_k , E_g , and *E*, respectively. An appropriate λ can be calculated by enforcing the *optimizability constraint*.

By increasing λ , the impact of E_g is increased, which has the effect of introducing more nearby scales to suppress rings for the final decomposition result. There exists a constraint such that rings are suppressed by introducing minimum amount of E_g , which makes the objective function locally optimizable, as shown by the solid line in Fig. 1(b). We call this constraint the *optimizability constraint* and it requires that the objective function has small gradients for small σ . This allows us to automatically calculate the most appropriate λ .

Given a relatively small value of σ , and denoting the derivatives for E_k and E_g at this value as E'_k and E'_{φ} , λ can be calculated as

$$\lambda = \frac{\epsilon - mE_k^{m-1}E_k'}{m(E_g^{m-1}E_g' - E_k^{m-1}E_k')},$$
(6)

where ϵ is a small gradient tolerance value which is selected to be around 0.001 in our experiments.

C. Solution procedure

With λ calculated, we can search for the filter parameter σ by minimizing the objective function (5). Note that by automatically computing λ , we ensure that the objective is always optimizable. Thus, the minimization can be quickly solved using Brent's method,¹⁷ which makes our optimization very stable and ensures that it always yields a solution. Once we obtain an optimized σ , we can perform fast Fourier transform (FFT) filtering to efficiently obtain scale decomposition results.

In order to avoid the wave reflection effects due to non-periodic boundary, a periodic reconnection by mirror extension of the data is performed, which is also required in curvelets.¹⁵ The procedure for KoSCO algorithm can be summarized as follows:

- A filter is designed in Kolmogorov spectrum consistent space with an unknown parameter σ , and is subsequently transformed to Fourier space.
- Based on this transformed filter in Fourier space, an objective function is constructed with Eq. (5).
- With the derivatives of E_k and E_g , λ is computed according to Eq. (6).
- Using the computed λ and given an initial σ (e.g., $\sigma_0 = 1.0$), the Brent's algorithm is employed to efficiently find the optimal filter parameter σ_m .
- Finally, FFT filtering is performed with parameter σ_m to get the scale decomposition result.

III. RESULTS AND DISCUSSIONS

A. Gibbs ringing artifacts

We use a circle image (see Fig. 2(a)) as a test case to demonstrate the performance of KoSCO compared to other existing methods with the same scale parameters (the location and bandwidth of the scale). The reason we chose this image is because it is simple with sharp edges, and hence may have strong Gibbs rings around the edge if inappropriately decomposed. Fig. 2(b) shows the scale decomposition result from a perfect BPF in Fourier space, which introduces strong ringing artifacts. Due to the global-support Fourier basis, a perfect BPF in Fourier space lacks suitable basis functions to completely represent structures, making the result contaminated by oscillating patterns.

Curvelets use local-support basis to avoid ringing artifacts. Fig. 2(c) shows scale decomposition result using curvelets (DCuT)⁹ which is the technique used by several groups recently. We can easily notice that it has less rings than perfect BPF in Fourier space. However, the rings are still not adequately suppressed.

In Fig. 2(d), we show the scale decomposition result from KoSCO. Clearly, the Gibbs rings are much more suppressed. Although we are unable to completely remove the Gibbs rings, KoSCO is still at an advantage when compared with the state-of-the-art. Eliminating rings when performing a multiscale decomposition is crucial since they can lead to spurious structures which may not physically exist. Turbulent flows are highly fluctuating and it is difficult to distinguish the presence of such rings from the true data.

B. Multi-scale diagnostics using KoSCO

In order to demonstrate the multi-scale decomposition capabilities of KoSCO, tests were conducted on a fractal image. Fractals can provide a very good example for multi-scale phenomena exhibiting the characteristic self-similarity. In Fig. 3, we show multi-scale decomposition results for the same fractal image (512×512) used by Bermejo-Moreno and Pullin.¹⁰ Perfect BPF produces spurious structures due to Gibbs ringing which makes the multi-scale decomposition contaminated by artifacts as can be seen in the first row. Using curvelets, one can get a maximum of 6 scales. The results by Bermejo-Moreno and Pullin¹⁰ can be seen in the second row of Fig. 3. We ignore the largest and the smallest scales because they do not convey meaningful structures for comparison. The largest scale is too coherent and shows almost the mean value, while the smallest scale is too incoherent and shows only noise.

Based on the spectrum of the results from curvelets, we estimate their locations and bandwidths and perform the decomposition again using KoSCO. Very similar decomposition results can be obtained as shown in the third row of Fig. 3. Even though we rely on Fourier transform, we have successfully suppressed Gibbs ringing artifacts. By observation, length scales from the decomposed results are decreasing in geometrical size as one goes from larger to smaller scales. In comparison to curvelets, our results show more continuous features and better preservation of original structures. For scale 2 in Fig. 3, at the center of the fractal, we see intermittent features for curvelets results, whereas for KoSCO, the features are smoother and therefore, constitute a better representation of the structures in the original image. Each decomposed scale is represented in the spectrum correspondingly as shown in Fig. 5(a). The results from KoSCO have sharper fall-offs than the results



FIG. 3. Comparison of multiscale decomposition results using perfect BPF, curvelets and KoSCO for a Julia fractal. Second row reprinted with permission from I. Bermejo-Moreno and D. I. Pullin, J. Fluid. Mech. **603**, 101 (2008). Copyright 2008, Cambridge University Press.

from curvelets and hence are better representations of the scales. With the help of optimization, we obtain optimal selection of fall-offs with suppressed Gibbs rings. The reduction of fall-off ensures that the decomposition results belong to an individual scale and reduce the inclusion of other scales. This can be attributed to the monotonicity criterion used while designing our filter (see Sec. II A), whereas in curvelets decomposition, the filter fall-offs tend to be fluctuating. Such fluctuating filters may result in intermittent features and may not preserve the scale structures well. On the other hand, our constraint on monotonicity helps to obtain more reliable results. Note that while curvelets are capable of producing only 6 scales, KoSCO has the capability to yield infinite number of scales.

C. Application to DNS data

We use forced isotropic turbulence data provided by John Hopkins University (JHU) turbulence data cluster¹⁸ at $Re_{\lambda} = 433$ to obtain scale decomposition results using KoSCO. The domain is a cube of length 2π with periodic boundary conditions having 512³ grid points.

For the 2D case, we use a cross section of this data at the mid-point of the Z-plane. Results for 8 different scales of velocity magnitude are shown in Fig. 4, and their corresponding spectrums are illustrated in Fig. 5(b). The different scales educe structures corresponding to different length scales.

Next, we decompose the 3D flow into 15 scales to demonstrate the ability to have continuous scale decomposition using KoSCO. The corresponding energy spectrum can be seen in Fig. 5(c). In order to visualize this volumetric data, iso-contouring technique is used. Iso-surfaces for this 3D decomposition can be seen in Fig. 6 where we show odd numbered scales for the sake of clarity. The iso-contour value used for visualizing the volumetric data has been chosen as the mean plus 1.5 times the standard deviation.^{10,16} It can be clearly seen that structures with reduced sizes are educed with increasing scale numbers.

Scales 1–3 are in the forcing range with the energy containing scales which comprise the large scale structures. Ellipsoidal-shaped structures can be observed for this scale. Scales 4–8 correspond



FIG. 4. 2D results for isotropic turbulence, (top left) Original flow, increasing scales from left to right and top to bottom (Red shows higher values and blue shows smaller values of the normalized velocity magnitude).

to the inertial range where tube-like structures are pre-dominant. The tubes become thinner as we go to smaller scales indicating a stretching process in the inertial region. For greater clarity of structures, readers are kindly suggested to look at video in Fig. 7 (Multimedia view). The dissipative range of scales can be seen in Scale 9–15 where structures are quite small and not visually recognizable.

The distribution of velocity magnitude at different scales normalized with its standard deviation is shown in Fig. 8. We only show odd-numbered scales to avoid confusion. It gives us the insight about the distribution for each of these scales in comparison to the original velocity field. Scales 4–8 lie in the inertial range and collapse on top of each other. For the dissipative range, i.e., Scales 9–15, the distribution function shows a reduced range. Hence, we can clearly distinguish between the inertial and dissipative range and these observations are inline with those reported by Bermejo-Moreno and Pullin.¹⁰ Note that curvelets can provide only 6 scales for this data size.

Some characteristic velocity and length scales for isotropic turbulence can be computed based on the following:

$$\overline{u_i^2} = \frac{2}{3} \int_0^\infty E_i(k) \mathrm{d}k,\tag{7}$$

$$L_i = \frac{\pi}{2\overline{u^2}} \int_0^\infty \frac{E_i(k)}{k} \mathrm{d}k,\tag{8}$$



FIG. 5. Spectrum for decomposition results. Black solid: original. For (a), light grey: curvelets, dark grey: KoSCO; for (b) and (c), grey dashed: odd scales using KoSCO; grey solid: even scales using KoSCO. (a) Julia fractal, (b) 2D isotropic flow, (c) 3D isotropic flow.

$$L'_{i} = \frac{\pi}{2\overline{u_{i}^{2}}} \int_{0}^{\infty} \frac{E_{i}(k)}{k} \mathrm{d}k, \tag{9}$$

where $E_i(k)$ and $\overline{u_i^2}$ are energy spectrum and squared characteristic integral velocity, respectively, for scale *i*, and η is the Kolmogorov length scale. Also, the following relations hold for the original velocity field E(k) and $\overline{u^2}$,

$$E(k) = \sum_{i} E_i(k), \tag{10}$$

$$\overline{u^2} = \sum_i \overline{u_i^2},\tag{11}$$

which simply describes that after summing up the individual scales, we recover the original velocity field. The corresponding values are shown in Table I for both the original velocity field and for different scales. This breakdown of scales show reasonably correct trends with reducing length and velocity scales as one goes from the mean and inertial to the dissipative ranges and how they are compared with the original velocity field.

We also perform a study on the impact of different resolutions on the estimation of the filter shape for 3D isotropic case.

We also perform a study on the influence of different resolutions on the estimation of the filter shape for 3D isotropic case as shown in Table II. The start of the band and the bandwidth along with



FIG. 6. Iso-contours for 3D isotropic case obtained using KoSCO.



FIG. 7. Video showing multi-scale turbulence structures for 3D isotropic case using KoSCO. (Multimedia view) [URL: http://dx.doi.org/10.1063/1.4871106.1]

parameter σ used for filter design (see Eq. (1)) is enlisted. As can be seen, the optimal σ value does not have any direct dependence on the resolution, scale location and scale bandwidth, and should be computed independently given any dataset.

The complete solution process for each scale requires 10-20 iterations for the optimization procedure. For a single scale decomposition of 512^3 dataset, it takes about 20 min on a desktop PC with Intel i7 processor. This is faster in comparison to curvelets with regards to the processing time. In addition, curvelets require storage of extra dimensions such as direction and orientation which can pose immense memory problems for high resolution DNS datasets. Hence, KoSCO performs scale decompositions using lesser computational resources as compared to curvelets.

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FIG. 8. Probability distribution of velocity magnitude for different scales.

TABLE I. Characteristic integral velocity and length for different scale	ocity and length for different scales.	veloci	Characteristic integra	TABLE I.
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Scale	$\overline{u_i^2}/\overline{u^2}$	L_i/η	L_i'/η	
Original	1	122.5405	122.5405	
1	0.333676	56.9197	170.5839	
2	0.251062	31.0462	123.6593	
3	0.182859	15.5049	84.7914	
4	0.140593	8.5200	60.6002	
5	0.122404	5.3627	43.8115	
6	0.105685	3.3913	32.0890	
7	0.087093	2.0499	23.5373	
8	0.067492	1.1749	17.4077	
9	0.046771	0.6041	12.9155	
10	0.028451	0.2734	9.6111	
11	0.015230	0.1087	7.1350	
12	0.007394	0.0388	5.2500	
13	0.003910	0.0150	3.8262	
14	0.002433	0.0068	2.7978	
15	0.000918	0.0022	2.3794	

TABLE II. Filter parameters for a 3D isotropic case with different resolutions.

	Resolution	l	32 ³	64 ³	128 ³	256 ³	384 ³	512 ³
Scale	Band-start	Bandwidth	σ	σ	σ	σ	σ	σ
1	0.2	0.05	0.03463	0.0427	0.02801	0.02298	0.02409	0.02409
2	0.25	0.05	0.05149	0.02409	0.02409	0.03909	0.02298	0.02298
3	0.3	0.05	0.02409	0.02409	0.03075	0.03480	0.04270	0.02298
4	0.35	0.05	0.05149	0.02409	0.02967	0.02801	0.03084	0.03480
5	0.4	0.05	0.03123	0.02999	0.02801	0.02801	0.02801	0.02967
6	0.45	0.05	0.02801	0.03008	0.02801	0.02801	0.02801	0.02801
7	0.5	0.05	0.02801	0.02801	0.02965	0.02801	0.02801	0.02801
8	0.55	0.05	0.02801	0.02801	0.02964	0.02801	0.02801	0.02801
9	0.6	0.05	0.02801	0.02801	0.02801	0.02801	0.02801	0.02801
10	0.65	0.05	0.02539	0.02801	0.02801	0.02801	0.02801	0.02968
11	0.7	0.05	0.02801	0.02801	0.02801	0.02801	0.02801	0.02801
12	0.75	0.05	0.02801	0.02801	0.02801	0.02801	0.02801	0.02801
13	0.8	0.05	0.02801	0.02801	0.03490	0.02801	0.02801	0.02801
14	0.85	0.05	0.03007	0.03219	0.02409	0.04346	0.03293	0.03092
15	0.9	0.05	0.02801	0.02801	0.02409	0.02409	0.02409	0.02409

IV. CONCLUSION

A methodology for multi-scale decomposition (KoSCO) of flow data is developed, which is consistent with bandpass filtering in the Kolmogorov spectrum. It provides an alternative to curvelets and tries to overcome some of their inherent disadvantages. Optimization methods have been used for the first time to design such decomposition procedure. Unlike previous methods, the size and location of the various scales can be easily controlled. The method has been proven to be capable of adequately suppressing Gibbs ringing artifacts while preserving sharp bandpass filter properties. The scale decomposition results for Julia fractal and turbulent flow fields have been presented together with their spectrum and probability distribution functions for different scales. KoSCO can provide a framework for multi-scale analysis for geometric structure identification with specific applications to turbulent flows, atmospheric flows, multiphase flows, etc. It can be easily extended to any N-dimensional data with multi-scale phenomena where scales are defined with respect to a certain spectrum. In addition, KoSCO can also support scale decomposition for both isotropic and anisotropic data since we operate in Fourier space rather than in the spatial domain.

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