Reynolds number scaling in separated turbulent boundary layers

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Introduction

In a theoretical study of the inner region in a turbulent boundary layer (TBL) under an adverse pressure gradient (APG), we investigate scaling laws near the wall, a crucial concept in turbulence models. The scaling laws were extended to the case of separation.

Direct numerical simulation (DNS) of turbulent boundary layer flows has been performed. In the strongest APG, a separation bubble is created while the boundary layer is attached everywhere in the weaker APGs. The data from the simulations are compared with the theoretical findings.

Excellent agreement between theory and data from the direct numerical simulation is found in the viscous sub-layer, while a qualitative agreement is obtained for the overlap region.

The inner part of the boundary layer.

By applying an asymptotic matching procedure to the equations governing a TBL in an APG, the expressions shown to the right are derived.

Two velocity scales appear in the momentum equation:

Viscous: $u_\tau \equiv \sqrt{\nu \left|\frac{\partial u}{\partial y}\right|_{y=0}}$

Pressure gradient: $u_p \equiv \left(\nu \frac{1}{\rho}\frac{dP}{dx}\right)^{1/3}$

Definitions:

$u^+ \equiv u/u_\tau$ $\quad y^+ \equiv y u_\tau/\nu$ $\quad u^p \equiv u/u_p$ $\quad y^p \equiv y u_p/\nu$

$\lambda = (u_p/u_\tau)^3$ $\quad \gamma = u_\tau/u_p$

x: streamwise direction $\quad y$: normal direction

Expressions

- Attached layer
  - Viscous sub-layer

$$u^+ = y^+ + \tfrac{1}{2}\lambda(y^+)^2 \quad (1) \quad \Longleftrightarrow \quad u^p = \tfrac{1}{2}(y^p)^2 + \gamma^2 y^p \quad (2)$$

  - Overlap

$$u^+ = \frac{1}{\kappa}\left(\ln y^+ - 2\ln\frac{\sqrt{1+\lambda y^+}+1}{2} + 2(\sqrt{1+\lambda y^+}-1)\right) + B \qquad (3)$$

$$\Longleftrightarrow$$

$$u^p = \frac{1}{\kappa}\left(2\sqrt{\gamma^2+y^p} + \gamma\ln y^p - 2\gamma\ln(\sqrt{\gamma^2+y^p}+\gamma)\right) + C \qquad (4)$$

- Separated layer
  - Viscous sub-layer

$$u^+ = -y^+ + \tfrac{1}{2}\lambda(y^+)^2 \quad (5) \quad \Longleftrightarrow \quad u^p = \tfrac{1}{2}(y^p)^2 - \gamma^2 y^p \quad (6)$$

  - Overlap

$$u^+ = \frac{1}{\kappa}\left[2\sqrt{\lambda y^+ - 1} - 2\arctan\left(\sqrt{\lambda y^+ - 1}\right)\right] + B \qquad (7)$$

$$\Longleftrightarrow$$

$$u^p = \frac{1}{\kappa}\left[2\sqrt{y^p - \gamma^2} - 2\gamma\arctan\left(\sqrt{\frac{y^p}{\gamma^2} - 1}\right)\right] + C \qquad (8)$$

- In the limit of ZPG, $\lambda \to 0$ $(u_p/u_\tau \to 0)$:

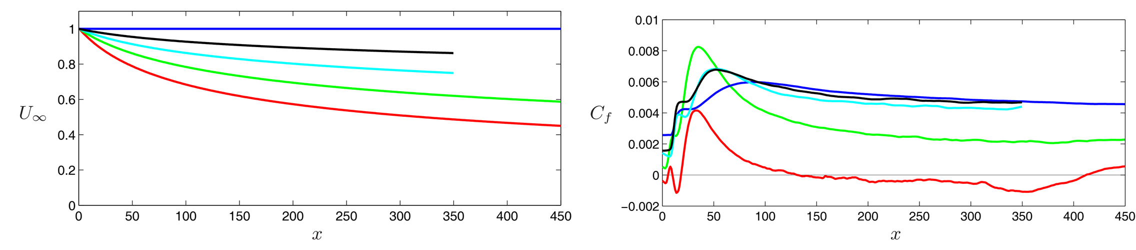
(1) $\to u^+ = y^+$ $\qquad$ (3) $\to u^+ = \frac{1}{\kappa}(\ln y^+) + B$

- At the point of separation, $\gamma \to 0$ $(u_\tau/u_p \to 0)$:

(2) $\to u^p = \frac{1}{2}(y^p)^2$ $\qquad$ (4) $\to u^p = \frac{1}{\kappa}2\sqrt{y^p} + C$

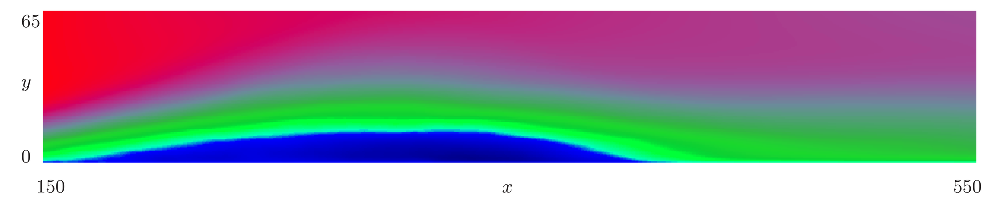
DNS

- Pseudo-spectral with Fourier series expansion in the wall parallel directions and Chebyshev series in the normal direction.
- The APG is imposed through the freestream velocity $U_\infty \sim x^m$ which is shown below together with the resulting skin friction.
- The Reynolds number corresponding to $x = 450$ is $Re_{\delta^*} = 920$ for the ZPG case.
- The simulation starts with a laminar boundary layer which undergoes transition as can be seen from C_f in the figure below.



Above: Results from five simulations are shown. In the zero pressure gradient case (ZPG), U_∞ is constant. Note that the gradual decreasing U_∞ between the different APGs give a strong effect on the skin friction (C_f). Only the strongest APG creates a separation bubble.

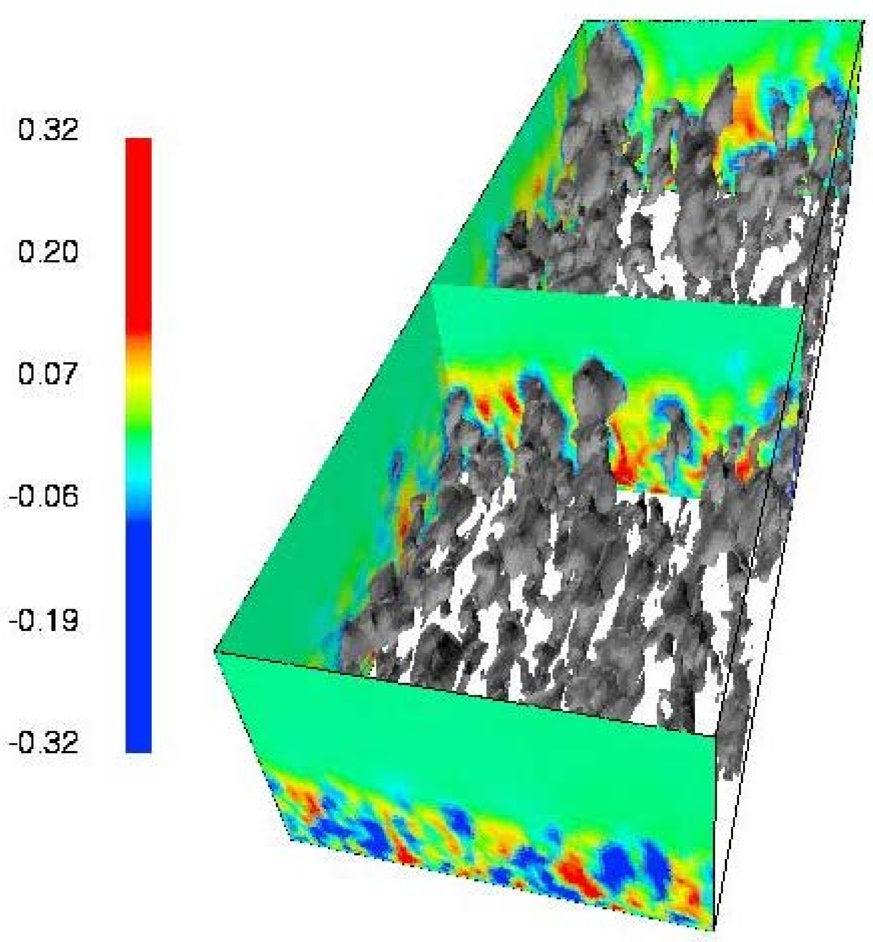
Below: The mean flow in the separated case is shown. The transitional region is not shown. The back-flow occurs in the blue region.



Results: **Instantaneous flow.**

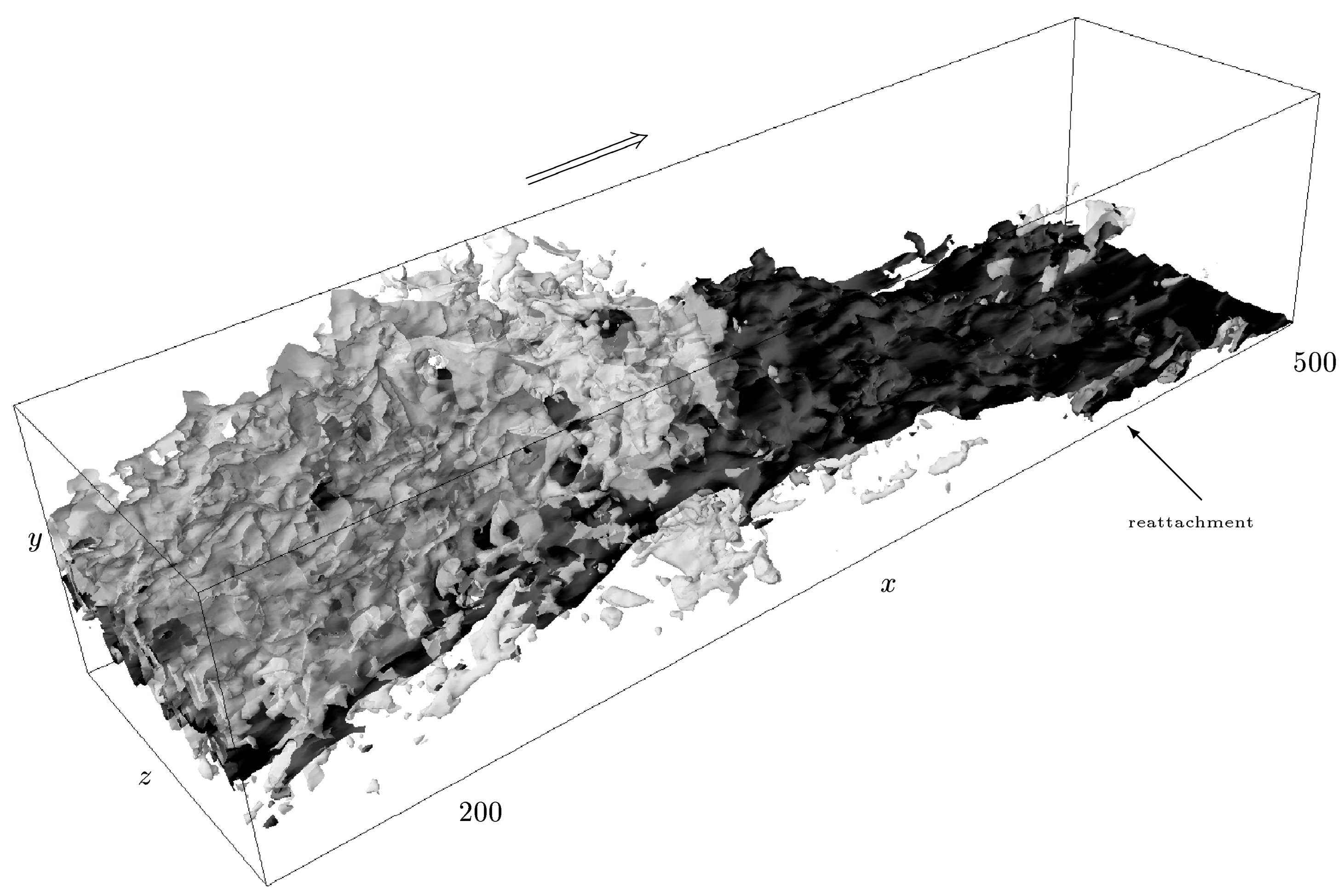
Attached flow

Weak APG $\quad$ Grey: low-speed streaks $\quad$ Color: streamwise disturbance velocity.



Separated flow

Light grey: positive normal velocity. $\quad$ Dark grey: positive streamwise velocity.

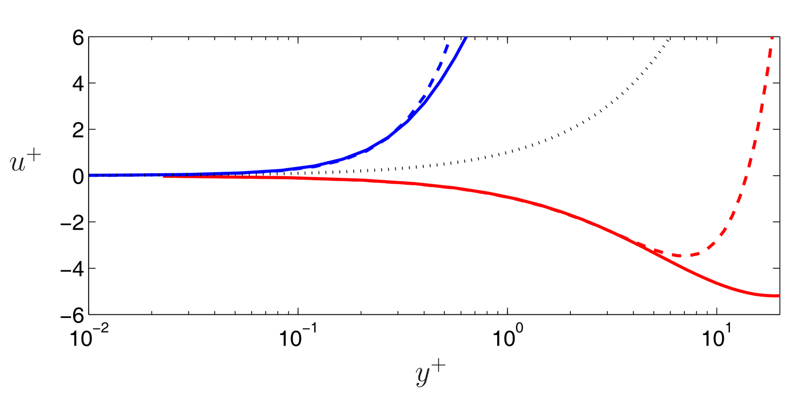


Results: **Velocity profiles.**

Before and after separation

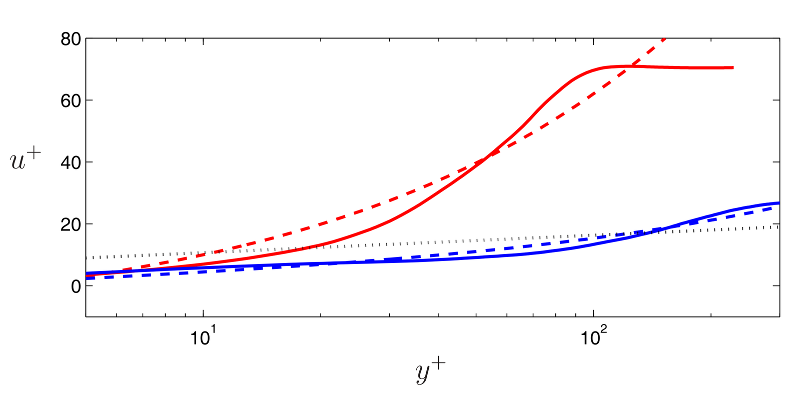
Viscous

In the separated region. — DNS data. - - equation (5).
In the attached region. — DNS data. - - equation (1).
$\cdots\ u^+ = y^+$.



Overlap

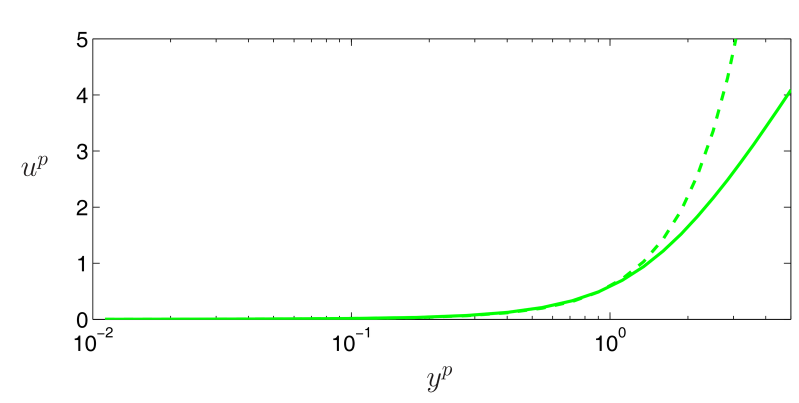
In the separated region. — DNS data. - - equation (7).
In the attached region. — DNS data. - - equation (3).
$\cdots\ u^+ = \frac{1}{\kappa}\ln y^+ + B$.



At the point of separation

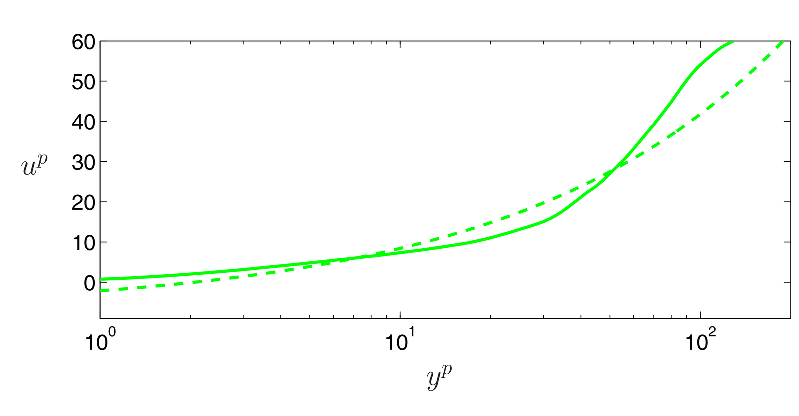
Viscous

— DNS data
- - $u^p = \frac{1}{2}(y^p)^2$.



Overlap

— DNS data
- - $u^p = \frac{1}{\kappa}2\sqrt{y^p} + C$.



Reynolds number dependence

If $U_\infty \sim x^m$, λ can be shown to decrease with increasing Reynolds number. Thus, the ZPG expressions will be valid for large enough Reynolds numbers.

At low Reynolds numbers however, λ also influences the velocity profiles for weak APGs.

When the APG is strong enough to induce separation, λ becomes infinite for any Reynolds number, and consequently the pressure gradient scaling must be used to avoid the singularity.

Conclusions

- For a zero pressure gradient TBL ($u_p = 0$), the viscous scaling yields a self-similar velocity profile $u^+(y^+)$.
- For a separating TBL ($u_\tau = 0$), the pressure gradient scaling yields a self-similar velocity profile $u^p(y^p)$.
- When $u_\tau \neq 0$ and $u_p \neq 0$, the two scalings are equivalent and the velocity profile is not self-similar.
- Excellent agreement between theory and data from the direct numerical simulation is found in the viscous sub-layer, even for the separated flow.
- A qualitative agreement is obtained for the overlap region. No perfect agreement can be expected due to the low Reynolds number at which the simulations were performed.